Course Code: BMG6DSE1B3
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Convex sets:

Let x be a point. If the point x can be expressed as

$$x = \lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n = \sum_{i=1}^{n} \lambda_i x_i$$

such that $\lambda_i \geq 0$ and $\sum \lambda_i = 1$ where $i = 1, \ldots n$ then x is said to be the convex combination of the points $x_1, x_2, \ldots, x_n$.

If we have two points x_1, x_2 then their convex combination is a point $x = \lambda_1 x_1 + \lambda_2 x_2$ such that $\lambda_1, \lambda_2 \geq 0$ and $\lambda_1 + \lambda_2 = 1$

Convex set: A set X is said to be a convex set if for any two points x_1, x_2 in the set, the line segment joining these two points also lie in the set.

So naturally if X is a convex set, then every point x can be expressed as

$$x = \lambda x_2 + (1 - \lambda) x_1, \quad 0 \leq \lambda \leq 1$$

where $x_1, x_2 \in X$ also lie on the set X.

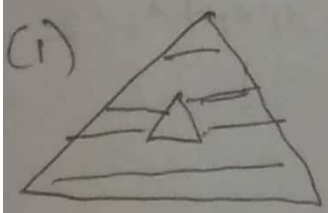
Example: A triangle in a xy plane is a convex set.

The figure mentioned below is not a convex set

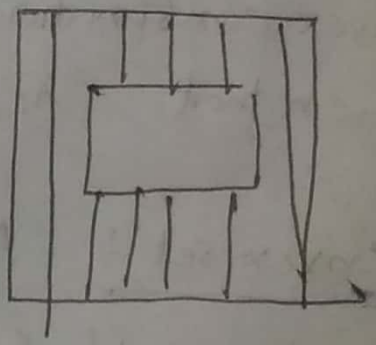
As the line segment joining PQ does not lie on the set.

Ex: Identify whether the following figures are convex set or not

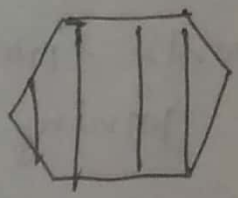
(i)



(ii)



(iii)



Show that $X = \{x : |x| \leq 2\}$ is a convex set.

Let $x_1, x_2 \in X$ then

$|x_1| \leq 2$, $|x_2| \leq 2$

$u = \lambda x_1 + (1-\lambda) x_2$ is the convex combination of x_1, x_2 if $1 \geq \lambda \geq 0$

So $|\lambda x_1 + (1-\lambda) x_2| \leq \lambda |x_1| + (1-\lambda) |x_2|$

$\leq 2\lambda + (1-\lambda) 2$

$= 2.$

So $u \in X$

Hence X is a convex set.